Light absorption in a two-dimensional Wigner crystal in high magnetic fields

1991 J. Phys.: Condens. Matter 3 9103

(http://iopscience.iop.org/0953-8984/3/46/011)

J. Phys.: Condens. Matter 3 (1991) 9103–9113. Printed in the UK

Light absorption in a two-dimensional Wigner crystal in high magnetic fields

S V Iordanskii and B A Muzykantskii
L D Landau Institute for Theoretical Physics, Kosygina 2, Moscow, USSR

Received 5 October 1990, in final form 30 April 1991

Abstract. It is shown that light absorption and light scattering have some additional structures owing to formation of bands for the magnetoexciton in a 2D Wigner crystal. The relative intensity of additional lines for absorption is rather small because of the necessity to violate the Kohn theorem. For light scattering there is no such restriction and the intensity of lines with momentum transfer to the Wigner lattice is rather large.

1. Introduction

Light absorption by 2D electrons in heterojunction or MOSFET devices in a strong magnetic field is intensively investigated. The most complete and rather clear physical picture is obtained for the fully occupied Landau level when absorption appears as a result of transitions on the next empty levels [1, 2]. The corresponding two-particle neutral excitation consisting of an electron and a hole is characterized by conserving momentum and some dispersion law defined by 2D electron–electron interaction. Various experiments on the absorption and scattering of light deals with the wide range of filling of Landau levels and it is interesting to determine the peculiarities of these processes for the case of the 2D Wigner crystal consisting of electrons or holes with a large lattice spacing. This is quite real because the Wigner crystal probably exists only for sufficiently low densities of crystallizing particles and low temperatures. In this paper we assume zero temperature. The considerable interest in such a problem is connected with experimental claims about the existence of the Wigner crystal in such a situation [3]. The first observation of the Wigner crystal (in the absence of a magnetic field) have been done in the system of electrons above a helium surface more than a decade ago [4].

The calculations for the form of line for light absorption near the cyclotron frequency were performed for 2D systems in a number of papers (see, e.g., [5, 6]) mostly using computers in various approximations to take into account electron–electron interaction. In a recent paper [7] the same problem was investigated directly for the Wigner crystal by considering the interaction of Wigner phonons with phonons of the underlying crystal lattice. The main difference between our work and that in [7] consists in taking into account the whole dispersion curve for magnetic excitons which splits in separate bands owing to the periodicity of the Wigner crystal. In [7], only the vicinity of zero magnetoexciton momentum was considered. In that case, all peculiarities connected with these bands vanish.

0953-8984/91/469103 + 11 $03.50

2. Electron–electron interaction and the Kohn theorem

The well known Kohn theorem states that the centre-of-mass motion does not depend on the interaction of electrons in a quadratic dispersion law manner if there are no external forces. Corresponding energy levels are quantized in units of free-electron cyclotron frequency ω_c. In this case the operator $\Sigma_k b_k^+ a_k$ for the creation of a magnetoexciton with zero momentum commutes with the Hamiltonian. Here we use the Landau gauge with k as the index of the state, a_k the annihilation operator on the lowest Landau level $n = 0$, and b_k^+ the creation operator on the first Landau level $n = 1$. It is essential to write down this commutation explicitly in order to see various effects in the heterojunctions invalidating the Kohn theorem and defining the peculiarities of light absorption.

We assume that the Coulomb energy $V_C(l_H) = e^2/\chi l_H$ is small compared with the free-electron cyclotron frequency, where χ is the dielectric constant of the underlying lattice and $l_H^2 = c\hbar/eH$ is the magnetic length. Accordingly we shall retain in the interaction Hamiltonian only the terms conserving the occupation of Landau levels:

$$H_{\text{int}} = \sum_{k_1,k_2} \int \frac{d^2q}{2\pi} \exp[iq_x(k_2 - k_1)] \left[V_{00,00}(q) a^+_{k_1+q_y/2} a^+_{k_2-q_y/2} a_{k_2+q_y/2} a_{k_1-q_y/2} \right.$$
$$\left. + [V_{00,11}(q) - \tilde{V}_{01,01}(q)] a^+_{k_1-q_y/2} b^+_{k_2-q_y/2} b_{k_2-q_y/2} a_{k_1-q_y/2}\right] \tag{1}$$

where

$$V_{00,00} = V(q)|L_{00}|^2 \qquad V_{00,11} = V(q)L_{00}L_{11} \qquad V_{01,01} = V(q)|L_{0,1}|^2 \tag{2}$$

$$L_{i,n}(q) = \int d\xi \exp(iq_x\xi)\Phi_i\left(\xi - \frac{q_y}{2}\right)\Phi_n\left(\xi + \frac{q_y}{2}\right)$$
$$\tilde{F}(q) = \int \frac{d^2p}{2\pi} \exp(iq \cdot p) F(p) \tag{3}$$

where $V(q)$ is the Fourier transform of the electron–electron interaction and the normalized wavefunctions of the Landau states have the form

$$\exp(ikx)\Phi_i(y + k)$$

with x, y coordinates in the 2D plane; we assume also that $l_H = 1$. The appearance of $\tilde{V}_{01}$ is connected with the possibility of representing the exchange as a direct interaction using some Fourier transformation [2].

The operator of the creation of a magnetoexciton with momentum p is [2]

$$A^+(p) = \sum_k \frac{1}{\sqrt{N_e}} \exp(-ip_x k) b^+_{k+p_y/2} a_{k-p_y/2}$$

where we insert the normalizing factor to ensure that $\langle A(p)A^+(p)\rangle = 1$ and N_e is the number of 2D electrons. This operator must be commuted with the Hamiltonian (1). After some simple but tedious algebra we obtain

$$i\frac{\partial A^+(p)}{\partial t} = [HA^+(p)]$$
$$= \varepsilon(p)A^+(p) - \frac{1}{2}\int \frac{d^2q}{(2\pi)^2}[V_{\text{eff}}(p,q) - \tilde{V}_{\text{eff}}(p,q)]A^+(p-q)\tilde{\rho}(q) \tag{4}$$

where $\tilde{\rho}$ is the Fourier transform of the hole density (the Wigner hole crystal) and the effective interaction potential is

$$V_{\text{eff}}(\boldsymbol{p},\boldsymbol{q}) = V_{00,00}(q)\exp(\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{q}/2) - V_{00,11}(q)\exp(\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{q}/2) - V_{01,01}(q)\exp(\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{q}/2) \tag{5}$$

$$\tilde{V}_{\text{eff}}(\boldsymbol{p},\boldsymbol{q}) = \tilde{V}_{00,00}(q)\exp(\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{q}/2) - \tilde{V}_{00,11}(q)\exp(\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{q}/2) - \tilde{V}_{01,01}(q)\exp(\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{q}/2). \tag{6}$$

For $\boldsymbol{p}=\boldsymbol{0}$ we get $V_{\text{eff}} = V(q)[L_{00}^2(q) - L_{00}(q)L_{11}(q) - |L_{01}|^2]$. The Kohn theorem requires that $V_{\text{eff}}(\boldsymbol{p}=\boldsymbol{0},\boldsymbol{q})$ and $\tilde{V}_{\text{eff}}(\boldsymbol{p}=\boldsymbol{0},\boldsymbol{q})$ vanish and indeed the combination of L_{ik} entering (4) vanishes for the quadratic dispersion law of electrons. In this case, Φ_i are oscillator wavefunctions and $L_{ik}(q)$ are connected with the Laguerre polynomial:

$$L_{00} = \exp(-q^2/2) \qquad L_{11} = (1-q^2/2)\exp(-q^2/2)$$
$$L_{01}^2 = (q^2/2)\exp(-q^2/2).$$

If we take into account non-parabolic corrections to the dispersion law which destroy exact compensation of L_{ik} in (5) and (6) there will be a non-zero $V_{\text{eff}} \sim q^2a^2/l_H^2$ where a is the atomic spacing of the order of the lattice constant for the underlying crystal. The vanishing of $V_{\text{eff}}(q=0)$ is connected with the general orthonormal properties of the eigenfunctions Φ_i.

We are interested in light absorption which is defined by the real part of the AC conductivity. Using standard perturbation theory in an electric field it is easy to express this (see, e.g. [2]) in terms of the retarded magnetoexciton Green function:

$$\mathrm{Re}[\sigma^+(\omega)] = (n_e e^2\omega_c/m)(n+1)\,\mathrm{Im}[G(\omega,p=0)] \tag{7}$$

where σ^+ is the conductivity for circular polarization at frequency ω, n_e is the number electron density, m is the effective mass and n is the number of the last occupied Landau level (the transition $n \to n+1$). The retarded Green function is defined as

$$G(\omega,\boldsymbol{p}) = \int_0^\infty \mathrm{d}t\,\langle\Psi_0|A(\boldsymbol{p})A^+(\boldsymbol{p})|\Psi_0\rangle\exp(\mathrm{i}\omega t)$$

where $|\Psi_0\rangle$ is the ground-state wavefunction for 2D electrons.

Because of the small density of particles participating in Wigner crystallization it is possible to use perturbation theory to calculate the magnetoexciton Green function using the equation of motion (4) and the simplest mean-field approximation. The corresponding diagram is shown in figure 1, where the thick black straight line denotes the retarded magnetoexciton Green function for filled Landau level:

$$G_0(\omega,\boldsymbol{p}) = 1/[\omega - \varepsilon(p) + \mathrm{i}\delta]$$

and

$$\varepsilon(p) = \hbar\omega_c + (e^2/2\chi l_H)\sqrt{\pi/2}[(1-\exp(p^2/4)[(1+p^2)I_0(p^2/4) - (p^2/2)I_1(p^2/4) + \sqrt{2/\pi}\,p\exp(p^2/2)]$$

is the magnetoexciton dispersion law for fully occupied Landau level [1] where I_n are Bessel functions of imaginary argument. The wavy lines correspond to $\tilde{V}_{\text{eff}} - V_{\text{eff}}$, the open circles denote the average $\langle\Psi_0|\tilde{\rho}|\Psi_0\rangle$ and the diagonal on the thick black straight line corresponds to $\mathrm{Im}[G_0(\omega,p)]$. Because of the periodicity of the Wigner crystal the

mean value of the hole density $\bar{\rho}(k)$ has δ-function singularities at k on the reciprocal lattice of the Wigner crystal.

The corresponding correction to the imaginary part of the magnetoexciton Green function is

$$\sum_{k_i} |V_{\text{eff}}(p=0,k_i) - \tilde{V}_{\text{eff}}(p=0,k_i)|^2 \, |\bar{\rho}(k_i)|^2 \left(\frac{1}{\omega-\omega_c}\right)^2 \text{Im}[G_0(\omega,k_i)] \tag{8}$$

where k_i are the vectors of the reciprocal Wigner lattice, and $\bar{\rho}(k_i)$ is the Fourier component of the hole density. Because non-parabolic corrections to the 2D electron dispersion law are not known with enough accuracy, we did not calculate the corresponding perturbation correction to the free-electron wavefunction for this diagram which otherwise will be zero. However, it is possible to obtain an estimation by the order of magnitude. The effective momentum transfer k_i to the Wigner lattice is of the order of inverse lattice spacing L^{-1} where $L \sim (1-\nu)^{-1/2} \gg 1$ (ν is the filling factor of last Landau level). From the orthonormal properties of Ψ_i it follows that $L_{ik} = \delta_{ik}$ and $V_{\text{eff}}(p=0) = 0$ in any case. Because $\tilde{V}_{\text{eff}} \sim e^2a^2/\chi l_H^3 \gg V_{\text{eff}}$ we get an estimate for correction to the imaginary part of Green function

$$(1-\nu)^2(a^4/l_H^4)[e^2/\chi l_H \hbar(\omega-\omega_c)]^2 \, \text{Im}[G_0(\omega,k_i)]. \tag{9}$$

For k_i of the order of basic reciprocal lattice vectors $\varepsilon(k_i) - \omega_c \sim (e^2/\chi)p \sim e^2/\chi L$ and we see that relative intensity of absorption on satellite will be proportional to the hole filling factor and is small as $(\delta\omega_c/\omega_c)^2(1-\nu)$ where $\delta\omega_c$ is the non-parabolic correction to cyclotron frequency.

Equation (8) can be generalized for any dynamical form factor S of the electron system:

$$\delta\{\text{Im}[G(\omega,p=0)]\} = \int \frac{d^2k}{(2\pi)^2}\frac{d\Omega}{2\pi}|V_{\text{eff}}(k) - \tilde{V}_{\text{eff}}(k)|^2 S(k,\Omega)\frac{1}{(\omega-\omega_c)^2}\text{Im}[G_0(\omega-\Omega,k)] \tag{10}$$

where

$$S(k,\Omega) = \int_{-\infty}^{\infty} \langle\bar{\rho}(t,k)\bar{\rho}(0,-k)\rangle \exp(i\Omega t)\, dt.$$

If the electron system is non-uniform, then there is a statistical part of $\tilde{S}(k,\Omega) = \delta(\Omega)\langle\tilde{\rho}_{st}(k)\tilde{\rho}_{st}(-k)\rangle$ which has δ-function singularities in k for the Wigner crystal. The real form of the absorption line can be obtained by integration taking into account some scattering process in $G_0(\omega,k)$, e.g. assuming the same scattering rate $1/\tau$ as for $\omega = \omega_c$, which can be taken from experimental data.

In the next section we consider other small corrections connected with crystal phonons and impurities which also violate the Kohn theorem and give rise to light absorption in the vicinity of the frequencies $\omega = \varepsilon(k_i)$.

3. The influence of phonons and impurities

Assuming that these effects are weak we shall use the lowest order of perturbation theory considering the simplest diagrams for Green functions shown in figure 2. The broken

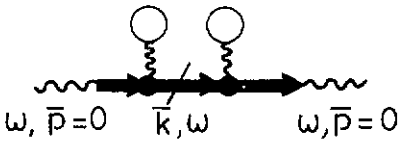

Figure 1. Simplest diagram for light absorption with momentum transfer to the Wigner crystal lattice due to non-parabolic corrections to the electronic dispersion law. The open circles correspond to a non-uniform electron density in the Wigner crystal. The thick black line is the free magnetoexciton propagator while the wavy line is the photon propagator.

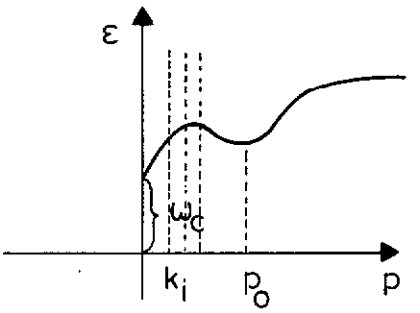

Figure 2. Dispersion curve for magnetoexcitons split into separate bands in the extended zones. Small gaps are not shown.

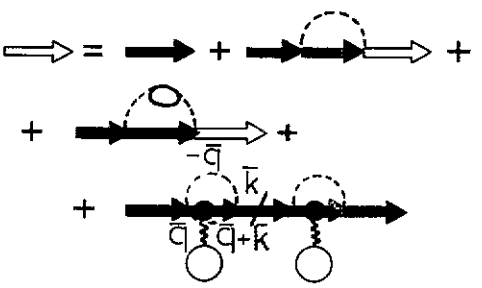

Figure 3. Light absorption with momentum transfer to the Wigner crystal due to 3d phonons. The broken curves represent the phonon propagator.

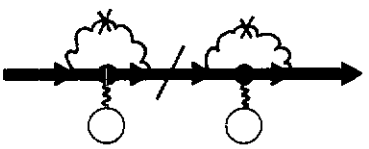

Figure 4. Light absorption with momentum transfer to the Wigner crystal due to impurities. The wavy lines with a cross are the correlation functions for the impurity potential.

lines correspond to the phonon propagator. According to the Kohn theorem there will be no interaction between the magnetoexciton with zero momentum and the Wigner crystallic field until virtual emission of phonons.

First we shall find corrections to the cyclotron frequency due to the first three diagrams in figure 3. It is easy to show that the main contribution is due to optical polarization phonons. The contribution of acoustical phonons with a piezoelectric effect is small:

$$4\pi/\chi^2(1/\chi_\infty - 1/\chi_0)^{-1}\, e_{14}/\rho s^2 \sim 10^{-2}$$

for GaAs, where ρ is the density, χ_∞ is the optical dielectric constant and e_{14} is the piezomodulus.

The electron–phonon interaction has a standard form and after integration over space coordinates the effective Hamiltonian for the magnetoexciton is

$$H_{\text{exc.ph}} = \sum_{p,q} g(p, q)\alpha^+(q)A^+(p - q_\parallel)A(p) + \text{cc} \tag{11}$$

where $\alpha^+(q)$ is phonon creation operator with 3D momentum q, $q_\parallel$ is 2D counterpart of phonon momentum, and

$$g(p,q) = [L_{11}(q_\parallel)\exp(\mathrm{i}[q_\parallel p]) - L_{00}(q_\parallel)\exp(-\mathrm{i}[q_\parallel p])][I(q_z)/q]g_0$$
$$g_0 = \mathrm{i}\sqrt{e^2\hbar\omega}\,(1/\chi_\infty - 1/\chi_0) \qquad I(p_z) = \int \mathrm{d}z\,|f_0(z)|^2\exp(\mathrm{i}q_z z). \tag{12}$$

Here $f_0(z)$ is the wavefunction for transverse motion (across the plane) for 2D electrons and ω_0 is optical phonon frequency. Assuming that $qa \sim a/l_H \ll 1$ where a is the transverse well dimension, we take $I(p_z) \sim 1$. Using perturbation theory and expression for effective vertex (11) we obtain the shift of magnetoexciton energy as

$$\Delta\varepsilon(p=0) = -\frac{e^2 g_0^2}{l_H 2\pi}\int \frac{\mathrm{d}^2 p}{p_\parallel^2 + p_z^2}\frac{[L_{00}(p) - L_{11}(p)]^2}{\varepsilon(0) - \varepsilon(p) - \hbar\omega_0}. \tag{13}$$

Neglecting $\varepsilon(0) - \varepsilon(p) \sim e^2/\chi_0 l_H \ll \hbar\omega_0$ we obtain $\Delta\varepsilon(p=0) = (3\sqrt{\pi}\,e^2/16 l_H)\times(1/\chi_\infty - 1/\chi_0) \approx 5\times 10^{-3} e^2/l_H$ for GaAs.

Although this shift is rather small, it violates the Kohn theorem and is of some interest because it will oscillate with filling factor. The oscillation is connected with screening by 2D electrons and can be considered by inserting a polarization electron bubble into the phonon propagator in figure 3. Exact calculation of appropriate correction is rather difficult because we need to make some assumptions about the ground state of 2D electrons. Assuming a scale of about $e^2/\chi_0 l_H$ for energy it is possible to estimate the amplitude of the oscillating component:

$$\Delta_{\mathrm{osc}}\varepsilon(p=0) = [\nu(1-\nu)e^2/l_H](1/\chi_\infty - 1/\chi_0)\times \text{constant}$$

with an uncertain constant of the order of unity. The above corrections to the dispersion law of magnetoexcitons are small and insensitive to the periodic structure of the Wigner crystal.

The matrix element for absorption on $\varepsilon(k_i)$ will be defined by an analytical expression corresponding to the final line in figure 3:

$$\frac{1}{\omega - \omega_c}\int \frac{|g(q,0)|^2}{\omega - \varepsilon(q) - \omega_0(q)}\frac{\tilde{\rho}(k)\tilde{V}_{\mathrm{eff}}(k)}{\omega - \varepsilon(q+k) - \omega_0(q)}\frac{\mathrm{d}^2 q}{(2\pi)^2}\frac{\mathrm{d}^2 k}{(2\pi)^2}.$$

If we neglect $\omega - \varepsilon$ compared with ω_0, it is possible to get an analytical result for conductivity:

$$\mathrm{Re}[\sigma^+(\omega)] = \frac{1}{(\omega-\omega_c)^2}\frac{1}{\omega_0^2}[\Delta\varepsilon(p=0)]^2$$
$$\times \sum_{k_i}|\rho(k_i)\tilde{V}_{\mathrm{eff}}(k_i)|^2\,\mathrm{Im}[G(\omega,k_i)]\frac{n_e e^2\omega_c(n+1)}{2m\omega}. \tag{14}$$

Because k_i is small compared with l_H^{-1} or of the order of it, the estimate for the relative intensity with the umklapp process to that without will be approximately

$$\{\mathrm{Im}[G_0(\omega,k_i)]/\mathrm{Im}[G_0(\omega_c,0)]\}\,(1/(\omega-\omega_c)\omega_0)^2\,(e^2/l_H)^4$$
$$\times (1/\chi_\infty - 1/\chi_0)^2|\tilde{\rho}(k_i)|^2(1/\chi_0^2)$$
$$\sim [\Delta\varepsilon(0)/\omega_0]^2(1-\nu)\{\mathrm{Im}[G_0(\omega,k_i)]/\mathrm{Im}[G_0(\omega_c,0)]\}. \tag{15}$$

Assuming that $e^2/l_H\omega_0 \sim 1$ and $a^4/l_H^4 \sim 10^{-8}$ we get from (15) and (9) that the phonon contribution to the intensity of absorption is 10^2 more than the non-parabolic contribution but unknown numerical factors can reverse the estimate.

Equation (14) can be generalized in the same form as equation (8):

$$\mathrm{Re}[\sigma^+(\omega)] = \frac{n_e e^2 \omega_c (n+1)}{2m\omega}\left(\frac{\Delta\varepsilon(p=0)}{\omega_0}\right)^2 \times \int \frac{\mathrm{d}^2k}{(2\pi)^2}\frac{\mathrm{d}\Omega}{2\pi} S(k,\Omega)|V_{\mathrm{eff}}(k) - \tilde{V}_{\mathrm{eff}}(k)|^2 \,\mathrm{Im}[G_0(\omega-\Omega,k)]. \tag{16}$$

The treatment of impurities is very much analogous to phonons. We shall not consider the corrections to the dispersion curve but only the effect of impurities on absorption. The corresponding diagram is given in figure 4, where the crosses denote the averages over impurities. We shall consider impurities in the Born approximation by substituting in the final expression the full scattering amplitude. The analytical expression corresponding to this diagram is

$$\sum_{k_i}\left(\frac{n_0}{\omega-\omega_c}\right)^2 \int \frac{\mathrm{d}^2k'}{(2\pi)^2}\frac{|U(k')|^2}{[\omega-\varepsilon(k')][\omega-\varepsilon(k'+k_i)]}|\tilde{\rho}(k_i)|^2\,|V_{\mathrm{eff}}-\tilde{V}_{\mathrm{eff}}|^2 \times \int \frac{\mathrm{d}^2k''}{(2\pi)^2}\frac{|U(k'')|^2}{[\omega-\varepsilon(k''+k_i)][\omega-\varepsilon(k'')]}. \tag{17}$$

Here n_0 is the number density of impurities and $U(k)/k^2$ is the Fourier component of their potential acting on electrons. There is a contribution to the imaginary part of the exciton Green function when we are on the pole just after scattering on the impurity. Such a process gives rise only to the continuum part of absorption and will not be considered because it is not specific to the Wigner crystal.

We can estimate the integral by taking into account only the poles of the integrand, i.e. $\varepsilon(k') = \omega$ or $\varepsilon(k_i + k') = \omega$. The difference $\varepsilon(k') - \varepsilon(k' + k_i)$ is just the gap in the energy spectrum of exciton and is of the first order in the periodic Wigner crystal potential, i.e. of the order of $\tilde{\rho}\tilde{V}_{\mathrm{eff}}$ and we obtain as an estimate for absorption shifted in frequency due to the periodicity of the Wigner crystal

$$\mathrm{Re}[\sigma^+(\omega)] \sim \sum_{k_i}\left[\frac{1}{l(k_i)}\left|\frac{\partial\varepsilon}{\partial k}\right| k_i\left[\frac{1}{(\omega-\omega_c)}\right]^2 \mathrm{Im}[G(k_i,\omega)\right] n_e e^2\omega_c/m\omega$$

where the mean free path $l^{-1}(k) = n_0\sigma(k)$ with the cross-section in the Born approximation given by $\sigma = |U(k)|^2\, k/(\partial e/\partial k)^2$. The generalization of (17) to the general form of the dynamical form factor is straightforward as in equations (8) and (14) and is omitted here.

4. Inelastic light scattering

In experiments by Pinczuk *et al* [8] on light scattering in the heterojunction a shift of the light frequency to the value of the so-called roton gap was observed. Let us consider the situation for the Wigner crystal. The frequency of scattered light in these experiments

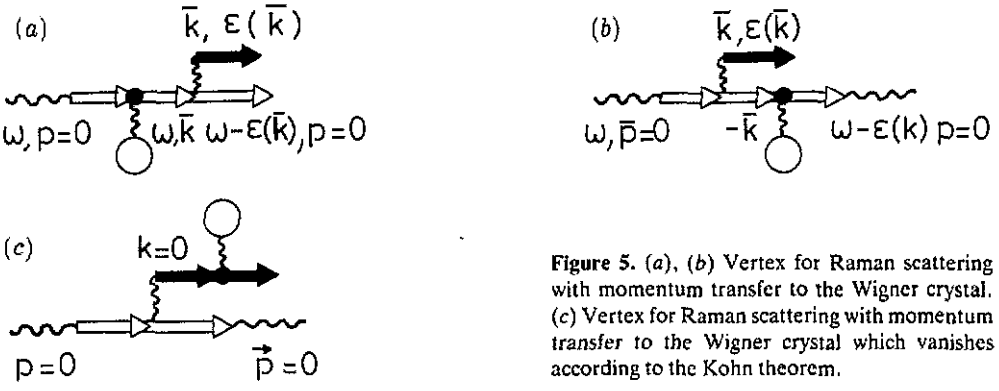

Figure 5. (*a*), (*b*) Vertex for Raman scattering with momentum transfer to the Wigner crystal. (*c*) Vertex for Raman scattering with momentum transfer to the Wigner crystal which vanishes according to the Kohn theorem.

was near the band gap for GaAs. The electron from the valence band on a Landau level goes to another Landau level in the conductance band, forming a virtual magnetoexciton with zero momentum. Then this exciton is scattered by the periodic potential of the Wigner crystal and changes its momentum on some vector of the reciprocal Wigner lattice. This virtual exciton can create in the Wigner crystal an additional exciton consisting of an electron on an unoccupied Landau level and a hole on an occupied Landau level in the conduction band. The corresponding diagrams are given in figures 5(*a*) and 5(*b*). All other diagrams such as that in figure 5(*c*) give zero contribution because of the Kohn theorem (a magnetoexciton with a zero momentum does not interact with Wigner crystals).

The non-zero contribution in figures 5(*a*) and 5(*b*) arises because the valence and conductivity bands have quite different dispersion laws, and holes and electrons created by light are in the same Landau states.

In order to compute the contribution in figures 5(*a*) and 5(*b*) to the scattering amplitude, we need the corresponding vertices. We must take into account the different natures of the Bloch functions in the valence and the conduction bands. The vertex for light interaction is defined by the integral of the current density:

$$H_i = \frac{e}{c} A \int \mathrm{d}^2x \frac{\mathrm{i}\hbar}{2m}\left(\frac{\partial \psi^+}{\partial x}\psi - \psi^+ \frac{\partial \psi}{\partial x}\right)$$

where the electronic operator is split into the valence and conduction counterparts:

$$\psi = \psi_c + \psi_v = \sum_k \left[u_{c,-(e/c)A} \exp(\mathrm{i}ky)\Phi_n(x+k) + u_{v,-(e/c)A} \exp(\mathrm{i}ky)\Phi_n(x+k)\right]$$

and u_c and u_v are the Bloch functions corresponding to the conduction and valence bands, respectively (see, e.g., [9]). It is easy to show that the main term in H_i is $H_i = (e/c)\boldsymbol{A}\cdot\boldsymbol{M}_{c,v}\Sigma_k \beta^+(k)\alpha(k)$ where the interband integral is

$$\boldsymbol{M}_{c,v} = -\mathrm{i}\frac{\hbar}{m}\int u_c^* \frac{\partial u_v}{\partial x}\mathrm{d}^2x$$

and the transitions are mainly between the same Landau levels for electrons and hole, $\alpha(k)$ is the operator for a hole in the valence band and $\beta(k)$ is the creation operator for an electron on the same empty Landau level m in the conduction band.

We are interested in the computation of the vertices in figure 5; therefore we shall neglect those terms in the Hamiltonian which shift the energy of the interband exciton:

$$H_{\mathrm{int}}^{(1)} = \sum_k \int \frac{\mathrm{d}^2 q}{(2\pi)^2} \exp[\mathrm{i} q_x (k_1 - k_2)$$
$$\times V(q)[L_{vm,vm}(q) L_{c0,c1}(q) \alpha^+_{k_1+q_y/2} b^+_{k_2-q_y/2} a_{k_2+q_y/2} \alpha_{k_1-q_y/2} + L_{cm,cm}(q)$$
$$\times L_{c0,c1}(q) \beta^+_{k_1+q_y/2} b^+_{k_2-q_y/2} a_{k_2+q_y/2} \beta_{k_1-q_y/2} + L_{cm,c0}(q) L_{cm,c1}(q)$$
$$\times \beta^+_{k_1+q_y/2} b^+_{k_2-q_y/2} \beta_{k_2+q_y/2} a_{k_1-q_y/2} + \mathrm{cc}]$$

where a_k corresponds to a hole in the lowest conduction Landau level, b^+_k corresponds to an electron in the first conduction Landau level. In the first term the magnetoexciton is created by a valence hole, and in the last two terms it is created by a conduction electron. Also there are terms describing the interaction of the interband exciton with electrons on the partially filled lowest Landau level:

$$H_{\mathrm{int}}^{(2)} = \sum_k \int \frac{\mathrm{d}^2 q}{(2\pi)^2} V(q) \exp[\mathrm{i} q_x (k_1 - k_2)] [L_{vm,vm}(q) L_{c0,c0}(q)$$
$$\times \alpha^+_{k_1+q_y/2} a^+_{k_2-q_y/2} a_{k_2+q_y/2} \alpha_{k_1-q_y/2} + L_{cm,cm}(q) L_{c0,c0}(q)$$
$$\times \beta^+_{k_1+q_y/2} a^+_{k_2-q_y/2} a_{k_2+q_y/2} \beta_{k_1-q_y/2} + L_{cm,c0}(q) L_{c0,cm}(q)$$
$$\times \beta^+_{k_1+q_y/2} a^+_{k_2-q_y/2} \beta_{k_2+q_y/2} a_{k_1-q_y/2}]. \tag{18}$$

In order to get the appropriate vertices we must commute the Hamiltonian with the creation operator for the interband exciton, i.e. $\tilde{A}^+(q) = \Sigma_k \exp(\mathrm{i} q_x k)\, \beta^+_{k+q_y/2} \alpha_{k-q_y/2}$. Because the Landau states of a hole in the valence band and an electron in the conduction band forming the interband exciton must be the same, there is some cancellation of direct interaction and we obtain

$$[H^{(2)} \tilde{A}^+(0)] = \int \frac{\mathrm{d}^2 p}{(2\pi)^2} \tilde{A}(p) \tilde{V}_{cm,c0}(-p) \tilde{\rho}(p)$$
$$\tilde{V}_{cm,c0}(p) = \int \frac{\mathrm{d}^2 q}{2\pi} \exp(\mathrm{i}\, p \cdot q)\, V(q) |L_{cm,c0}(q)|^2. \tag{19}$$

For commutation with $H^{(1)}$ there is also some cancellation and we get

$$[H^{(1)} \tilde{A}^+(0)] = \int \frac{\mathrm{d}^2 p}{(2\pi)^2} \tilde{A}^+(p) A^+(-p) \tilde{V}_{c0,c1}(-p)$$

where

$$A^{(+)}(p) = \sum_k \exp(-\mathrm{i} k p_x)\, b^{(+)}_{k+p_y/2} a_{k-p_y/2}$$

is the creation operator for a magnetoexciton in the conduction band on the partially filled lowest Landau level and

$$\tilde{V}_{c0,c1}(p) = \int \frac{\mathrm{d}^2 q}{2\pi} \exp(\mathrm{i}\, p \cdot q)\, V(q) L_{cm,c0}(q) L_{cm,c1}(q).$$

The results obtained above can be reformulated as an effective Hamiltonian for the

interband magnetoexciton:

$$H_{\text{eff}} = \sum_p \left(E(p)\tilde{A}^{(+)}(p)\tilde{A}(p) + \int \frac{d^2p'}{(2\pi)^2} \tilde{A}^+(p')\tilde{A}(p)\tilde{V}_{cm,c0}(p-p')\tilde{\rho}(p-p') \right.$$
$$\left. + \int \frac{d^2p}{(2pi)^2} \tilde{A}^+(p')\tilde{A}(p)A(p-p')\tilde{V}_{c0,c1}(p-p') + \text{cc} \right) \tag{20}$$

where $E(p)$ is the interband magnetoexciton energy, the precise form of which is not essential.

From (20) it is easy to get the analytical expression for the inelastic scattering rate of light with the change in frequency $\omega \to \omega'$, corresponding to the umklapp process shown in figure 5(a):

$$T^u_{\omega\to\omega'} \sim \sum_{k_i} |M_{c,v}\tilde{V}_{cn,c(n+1)}(k_i)\tilde{V}_{cm,cn}(-k_i)\tilde{\rho}(k_i)|^2$$
$$\times \left| \frac{1}{[\omega - E(0) + i\delta][\omega - E(k_i) + i\delta][\omega - E(0) - \varepsilon(k_i) + i\delta]} \right|^2 \text{Im}[G(\omega - \omega', k_i)] \tag{21}$$

where we have omitted some constant factors which are the same also for light scattering without umklapp processes:

$$T^0_{\omega\to\omega'} \sim \frac{|M_c v|^2 |\tilde{V}(0)|^2}{|\omega - E(0)|\,|\omega' - E(0)|} \text{Im}[G(\omega - \omega', 0)].$$

We see that the relative intensity will be of the order of

$$|\tilde{\rho}(k_i)\tilde{V}(k_i)/[\omega - E(k_i)]|^2 \{\text{Im}[G(\omega - \omega', k_i)]/\text{Im}[G(\omega - \omega', 0)]\}$$
$$\sim |e^2/\chi_0 l_H[\omega - E(k_i)]|^2 \{\text{Im}[G(\omega - \omega', k_i)]/\text{Im}[G(\omega - \omega', 0)]\}(1-\nu)^2.$$

The estimate of the analytical expression for figure 5(b) does not differ essentially.

We see that violation of translational symmetry in the Wigner crystal gives rise to light scattering shifted by the magnetoexciton energy for momentum on the reciprocal lattice with quite a large intensity. If the momentum transfer to the Wigner lattice is large compared with basic reciprocal vectors, the peaks in $\tilde{\rho}(k_i)$ will be small and smooth and we shall see only a maximum in the intensity of scattered light in the vicinity of the roton gap where the density of states is the largest.

Some similar process will take place even in electronic liquid, probably with some additional energy loss due to momentum transfer to the liquid. Equation (21) can be generalized for any kind of dynamical form factor for a 2D electronic liquid as was done previously for light absorption:

$$T^u_{\omega\to\omega'} \sim \int \frac{d\varepsilon}{2\pi} \frac{d^2k}{(2\pi)^2} |M_{c,v}\tilde{V}_{cn,c(n+1)}(k)\tilde{V}_{cm,cn}(-k)|^2 S(k, \omega - \varepsilon - \omega')$$
$$\times \frac{\text{Im}[G(\varepsilon, k)]}{|[\omega - E(0)][\omega' - E(k)][\omega' - E(0)]|^2}. \tag{22}$$

The dynamical form factor for an electronic 2D system at arbitrary filling of the Landau level is in fact not known. At small densities the energy of intra Landau level excitations is also small. Experimental estimates give the value of the gap in fractional Hall states as small compared with $e^2/\chi_0 l_H$; therefore in the general expression for transition

probability $T^{u}_{\omega\to\omega'}$ the frequency ω' cannot probably deviate strongly from $\omega' = \omega - \varepsilon(k)$, and because of the large density of states near the roton gap the scattering will be concentrated near $\omega' = \omega - \Delta_{\text{roton}}$. These considerations can probably explain the observation by Pinczuk *et al* [8] of the violation of translational symmetry for light scattering in an extremely pure heterojunction.

Acknowledgments

The authors express their gratitude to E I Rashba for valuable discussions and to the Institute of Scientific Interchange (Torino, Italy) for partial support of this work.

References

[1] Bychkov Yu A, Iordanskii S V and Eliashberg G M 1981 *Zh. Eksp. Teor. Fiz. Pis. Red.* **33** 143 (Engl. Transl. 1981 *JETP Lett.* **33** 143)
[2] Kallin C and Halperin B I 1984 *Phys. Rev.* B **30** 5655; 1985 *Phys. Rev.* B **31** 3635
[3] Andrei E Y, Deville G, Glatti D C, Williams F I B, Paris E and Etiene B 1988 *Phys. Rev. Lett.* **60** 2765; 1988 *Phys. Rev. Lett.* **62** 1926
[4] Grimes C C and Adams G 1979 *Phys. Rev. Lett.* **42** 795
[5] Peeters F M, Wu Xiaguang and Devreese J T 1988 *Surf. Sci.* **196** 474
[6] Hu G Y and O'Connell R F 1989 *Phys. Rev.* B **40** 11 701
[7] Dyckman M 1982 *J. Phys. C: Solid State Phys.* **15** 7397
[8] Pinczuk A, Valladares J P, Heiman D, Gossard A C, English J H, Tu C W, Pfeifer L and West K 1988 *Phys. Rev. Lett.* **61** 2701
[9] Lifshits E M and Pitaevsky L P 1973 *Statistical Physics* vol 9 (Moscow: Nauka)